Consider the IVP $y' = 2y^2 - 3x, \quad y(1) = -2$. Use Euler's method with $h = 0.2$ to estimate $y(1.4)$. **SCORE: _____ / 4 PTS**

$$y(1.2) \approx y(1) + y'(1)(1.2-1) \quad 1 \quad 1$$

$$= \underline{-2 + (2(-2)^2 - 3(1))(0.2) = -2 + 5(0.2)} = \underline{-1}$$

$$y(1.4) \approx y(1.2) + y'(1.2)(1.4-1.2) \quad 1 \quad 1$$

$$\approx \underline{-1 + (2(-1)^2 - 3(1.2))(0.2) = -1 + (-1.6)(0.2)} = \underline{-1.32}$$

Determine if $y = A\sqrt{x} + \dfrac{B}{x^2} + \dfrac{x^2}{4}$ is a family of solutions of the DE $4x^2y'' + 10xy' - 4y = 5x^2$. **SCORE: _____ / 6 PTS**

**State your conclusion clearly.**

$$y = Ax^{\frac{1}{2}} + Bx^{-2} + \tfrac{1}{4}x^2$$

$$y' = \underbrace{\tfrac{1}{2}Ax^{-\frac{1}{2}} - 2Bx^{-3} + \tfrac{1}{2}x}_{} \quad 1$$

$$y'' = \underbrace{-\tfrac{1}{4}Ax^{-\frac{3}{2}} + 6Bx^{-4} + \tfrac{1}{2}}_{} \quad 1$$

$$4x^2y'' + 10xy' - 4y = \boxed{\begin{aligned} &-\cancel{A}x^{\frac{1}{2}} + 24\cancel{B}x^{-2} + 2x^2 \\ &+5\cancel{A}x^{\frac{1}{2}} - 20\cancel{B}x^{-2} + 5x^2 \\ &-4\cancel{A}x^{\frac{1}{2}} - 4\cancel{B}x^{-2} - x^2 \end{aligned}} \quad 1$$

$$= \underbrace{6x^2}_{2} \qquad \underbrace{\text{NO}}_{1}$$

In certain population models, a group will go extinct if and only if its population is below a certain level (called the **SCORE: _____ / 4 PTS** survival threshold $P_S$). Write a differential equation for the population of a group which is going extinct, if the rate of change of its population is proportional to the difference between the threshold and the existing population. **Justify your answer properly, but briefly.**
**NOTE: The signs of all constants should be stated clearly.**

$\frac{dP}{dt} = k(P - P_s)$ $2\frac{1}{2}$

GOING EXTINCT $\rightarrow$ $\frac{1}{2}$ $P < P_s$ AND $\frac{dP}{dt} < 0$ $\frac{1}{2}$

$P - P_s < 0$

SO $k > 0$ $\frac{1}{2}$

NOTE: THE BETTER ANSWER IS

$\frac{dP}{dt} = -k(P_s - P)$ SINCE IT IS

MORE "OBVIOUS" THAT $\frac{dP}{dt} < 0$

What does the Existence and Uniqueness Theorem tell you about possible solutions to the IVP **SCORE: _____ / 4 PTS**

$\frac{dy}{dx} = \frac{\sqrt[3]{y-2}}{x+6}, \quad y(8) = 2$ ? **Justify your answer properly, but briefly.**

$f = (y-2)^{\frac{1}{3}}(x+6)^{-1}$

$1\frac{1}{2}$

$f_y = \frac{1}{3}(y-2)^{-\frac{2}{3}}(x+6)^{-1}$ IS NOT CONTINUOUS IN AN OPEN REGION AROUND $(8,2)$ SINCE $f_y(8,2)$ DNE

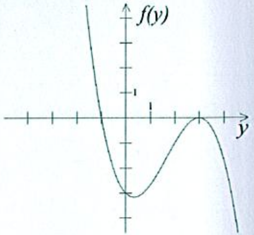
$1$

$= \frac{1}{3\sqrt[3]{(y-2)^2}\,(x+6)}$

E+U TELLS US NOTHING

$1\frac{1}{2}$

Consider the autonomous DE $\frac{dy}{dx} = f(y)$, where $f(y)$ is the function whose graph is shown on the right. **SCORE: _____ / 6 PTS**




[a] Find all equilibrium solutions of the DE and classify each as stable, unstable or semi-stable.
**You must draw a phase portrait to get full credit.**

$f(y) = 0 \longrightarrow y = -1,\ y = 3$ EQUILIBRIUM SOLUTIONS 1

1

SEMI-STABLE, 3

1

STABLE, -1

$\frac{dy}{dx} = f(y) < 0$

$< 0$

$> 0$

2

[b] If $y = m(x)$ is a solution of the DE such that $m(4) = 2$, what is $\lim_{x \to \infty} m(x)$ ?

$-1$, 1